Chapter 9 Review

Name Key

Determine whether each sequence is arithmetic or geometric. If possible, find the common difference or ratio.

1. 7, 14, 28, 56, 112, ...

geometric $r=2$

2. 25.5, 31, 36.5, 42, 47.5, ...

Arithmetic $d=5.5$

3. -3, 6, 21, 42, 69, ...

Neither

4. 2, 4, 6, 2, 4, ...

Neither

5. 15.5, 28, 40.5, 53, 65.5, ...

arithmetic

$d=12.5$

6. $4, 1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots$

geometric $r=\frac{1}{4}$

Find the first 5 teams of each sequence.

7. $a_1 = 16, a_n = 0.25a_{n-1}$

$16, 4, 1, \frac{1}{4}, \frac{1}{16}$

8. $a_n = 5(n-1)$

$0, 5, 10, 15, 20$

9. $a_n = 3^n - 4$

$-1, 5, 23, 77, 239$

10. $a_n = (n+2)^2$

$9, 16, 25, 36, 49$

Find the indicated term of the arithmetic sequence.

11. $a_1 = 4.5,\ \ d = 1.5,\ \ n = 8$

$a_8 = 4.5 + 1.5(8-1)$

$a_8 = 4.5 + 10.5 = 15$

12. $a_1 = 74,\ \ d = -6,\ \ n = 10$

$a_{10} = 74 + -6(10-1)$

$a_{10} = 74 + -54 = 20$

13. $a_3 = 29$ *and* $a_6 = 56$; $a_1 = ?$

Find d $\underbrace{29, \_, \_, 56}$ $3d = 27$

$d = 9$

$a_1 = 11$

14. $a_4 = 16$ *and* $a_7 = -2$; $a_1 = ?$

Find d $\underbrace{16, \_, \_, -2}$ $3d = -18$

$d = -6$

$a_1 = 34$

Find the indicated term of the geometric sequence.

15. $a_1 = 200,\ \ r = \frac{1}{2},\ \ n = 6$

$a_6 = 200 \cdot \left(\frac{1}{2}\right)^{6-1} = 200 \cdot \frac{1}{32}$

$a_6 = \frac{25}{4}$

16. $a_1 = -1,\ \ r = 3,\ \ n = 7$

$a_7 = -1 \cdot 3^{7-1} = -1 \cdot 3^6 = -729$

17. $a_4 = 4$ *and* $a_5 = 8$; $a_1 = ?$

$\overset{3r}{4, 8}$ $r = 2$

$a_4 = a_1 \cdot 2^{4-1}$ $a_1 = \frac{1}{8}$

$4 = a_1 \cdot 2^3$

18. $a_3 = 125$ *and* $a_5 = 5$; $a_1 = ?$

$125, \_, 5$ $r^2 = \frac{1}{25}$

$r = \frac{1}{5}$

$a_3 = a_1 \cdot \left(\frac{1}{5}\right)^{3-1}$

$125 = a_1 \cdot \frac{1}{25}$ $a_1 = 3125$

Find the missing terms in each arithmetic sequence

19. 13, 21, 29, 37

$3d = 24$

$d = 8$

20. 9.5, 7, 4.5, 2, -0.5

+d +d +d +d

-10

$4d = -10$

$d = -\frac{5}{2} = -2.5$

21. 10, 14, 18, 22, 26

+d +d +d +d

$4d = 16$

$d = 4$

22. 50, 41, 32, 23, 14

+d +d +d +d

$4d = 36$

$d = -9$

Find the missing terms in each geometric sequence

23. -16, -8, -4, -2

$\times\frac{1}{4}$

$r^3 = \frac{1}{8}$ $r = \frac{1}{2}$

24. -6, -12, -24, -48, -96

×r ×r ×r ×r

×16

$r^4 = 16$ $r = 2$

25. 5, -15, 45, -135

×r ×r ×r

×-27

$r^3 = -27$ $r^3 = -3$

26. 5, 10, 20, 40, 80

×r ×r ×r ×r

×16

$r^4 = 16$ $r = 2$

Write a simplified explicit formula for the nth term and find the 10th of each sequence.
You need to determine if it is arithmetic or geometric first.*

27. -4, -8, -12, -16, ...

Arithmetic $d = -4$

$a_n = -4 + -4(n-1)$

$a_{10} = -4 + -4(10-1)$

$a_{10} = -40$

28. 5, 20, 80, 320, ...

$r = 4$ Geometric

$a_n = 5 \cdot 4^{n-1}$

$a_{10} = 5 \cdot 4^{10-1}$

$a_{10} = 1,310,720$

29. -16, -8, -4, ...

geometric $r = \frac{1}{2}$

$a_n = -16 \cdot (\frac{1}{2})^{n-1}$

$a_{10} = -16 \cdot (\frac{1}{2})^{10-1} = -\frac{1}{32} = -.03125$

30. 27, 18, 12, ...

$r = \frac{2}{3}$

$a_n = 27 \cdot (\frac{2}{3})^{n-1}$

$a_{10} = 27 \cdot (\frac{2}{3})^{10-1}$ $a_{10} = .70233$

31. 27, 18, 12, ...

$r = \frac{2}{3}$

$a_{10} = .70233$

32. -6, 12, -24, ...

$r = 2$

$a_n = -6 \cdot 2^{n-1}$

$a_{10} = -6 \cdot 2^{10-1} = 3072$

Arithmetic $S_n = n\left(\frac{a_1+a_n}{2}\right)$ geometric $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ Infinite geometric $S = \frac{a_1}{1-r}$

Find S_n for each series described. You will need to determine if it's an arithmetic or geometric sequence. You may also need to find a_n.

31. 18, 21, 24, 27, ... n = 30 Arithmetic

$d = 3$ $a_{30} = -69$ $a_1 = 18$ $n = 30$

$S_{30} = 30\left(\frac{18 + -69}{2}\right)$

$S_{30} = -765$

32. $\frac{3}{4}$, 3, 12, 48, ... n = 6 geometric

$r = 4$ $a_1 = \frac{3}{4}$ $n = 6$

$S_6 = \frac{3}{4}\left(\frac{1-4^6}{1-4}\right) =$

$S_6 = 1023.75$

33. 4, $\frac{4}{5}$, $\frac{4}{25}$, $\frac{4}{125}$, ... infinite geometric

$S_\infty = \frac{4}{1-\frac{1}{5}}$ $r = \frac{1}{5}$ $a_1 = 4$ $n = \infty$

$S = 5$

34. 20, 18.5, 17, 15.5, ... n = 15 Arithmetic

$d = -1.5$ $a_{15} = -1$ $a_1 = 20$ $n = 15$

$S_{15} = 15\left(\frac{20 + -1}{2}\right)$

$S_{15} = 142.5$

35. -2700, 900, -300, ... infinite geometric

$S = \frac{-2700}{1-(-\frac{1}{3})}$ $r = -\frac{1}{3}$ $a_1 = -2700$ $n = \infty$

$S = -2025$

36. 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, ... n = 5 geometric

$r = \frac{1}{3}$ $a_1 = 1$ $n = 5$

$S_5 = 1\left(\frac{1-(\frac{1}{3})^5}{1-\frac{1}{3}}\right)$

$S_5 = 1.4938 = \frac{121}{81}$

37. 5, -15, 45, -135, ... n = 8 geometric

$r = -3$ $a_1 = 5$ $n = 8$

$S_8 = 5\left(\frac{1-(-3)^5}{1-(-3)}\right)$

$S_8 = 305$

38. -1, -5, -9, -13, ... n = 18 Arithmetic

$d = -4$ $a_{18} = -69$ $a_1 = -1$ $n = 18$

$S_{18} = 18\left(\frac{-1 + -69}{2}\right)$

$S_{18} = -630$

39. 13, 2, -9, -20, ... n = 18 Arithmetic

$d = -11$ $a_{18} = -174$ $a_1 = 13$ $n = 18$

$S_{18} = 18\left(\frac{13 + -174}{2}\right)$

$S_{18} = 18\left(\frac{161}{2}\right)$

$S_{18} = -1449$

40. 25, -5, 1, $\frac{-1}{5}$, ... infinite geometric

$r = -\frac{1}{5}$ $a_1 = 25$ $n = \infty$

$S = \frac{25}{1-(-\frac{1}{5})}$

$S = \frac{125}{6} = 20.833$

41. 1, 5, 25, 125, ... n = 6

$r = 5$ $a_1 = 1$ $n = 6$

$S_6 = 1\left(\frac{1-5^6}{1-5}\right)$

$S_6 = +3906$

42. 40, 30, 20, 10, ... n = 10 Arithmetic

$d = -10$ $a_1 = 40$ $n = 10$

$S_{10} = 10\left(\frac{40 + -50}{2}\right)$

$S_{10} = -50$

43. $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{9}$, ... infinite geometric

$r = \frac{2}{3}$ $a_1 = \frac{3}{4}$ $n = \infty$

$S = \frac{\frac{3}{4}}{1-\frac{2}{3}}$

$S = 2.25 = \frac{9}{4}$

44. 12, 7, 2, -3, ... n = 16 Arithmetic

$d = -5$ $a_{16} = -63$ $a_1 = 12$ $n = 16$

$S_{16} = 16\left(\frac{12 + -63}{2}\right)$

$S_{16} = -816$

45. -6, -1, 4, 9, ... n = 14 Arithmetic

$d = 5$ $a_{14} = 59$ $a_1 = -6$ $n = 14$

$S_{14} = 14\left(\frac{-6 + 59}{2}\right)$

$S_{14} = 371$

45. 12, -24, 48, -96, ... n = 6 geometric

$r = -2$ $a_1 = 12$ $n = 6$

$S_6 = 12\left(\frac{1-(-2)^6}{2}\right)$

$S_6 = -378$

Evaluate the given sum.

(arithmetic, geometric, constant, linear, and quadratic are all mixed together)

46. $\sum_{k=1}^{9}(5k+8) = \underset{a_1}{13} + \underset{a_2}{18} + \underset{a_3}{23} + \ldots + \underset{a_9}{53}$

Arithmetic

$a_1 = 13$

$a_9 = 53$ $\quad S_9 = 9\left(\frac{13+53}{2}\right)$ $\quad S_9 =$

$n = 9$

47. $\sum_{k=1}^{20}(-2.75k+15) = \underset{a_1}{12.25} + 9.5 + \ldots + \underset{a_{20}}{-40}$

$a_1 = 12.25$ $\quad S_{20} = 20\left(\frac{12.25 + -40}{2}\right)$

$a_{20} = -40$

$n = 20$ $\quad S_{20} = -27\cancel{7}.5$ Arithmetic

48. $\sum_{1}^{5} 12(2)^{k-1} = \underset{a_1}{12} + 24 + 48 + \ldots + \underset{a_5}{192}$

$a_1 = 12$

$r = 2$ $\quad S_n = 12\left(\frac{1-2^5}{1-2}\right) = 372$

$n = 5$

49. $\sum_{k=1}^{7}(-4)^{k-1} = 1 + -4 + \ldots + 4096$

$a_1 = 1$

$r = -4$ $\quad S_7 = 1\left(\frac{1-(-4)^7}{1-(-4)}\right) = +819$

$n = 7$

50. $\sum_{k=1}^{8} -5 = \underset{a_1}{-5} + -5 + -5 + \ldots + \underset{a_n}{-5}$

$a_1 = -5$

$a_8 = -5$ $\quad S_8 = 8\left(\frac{-5+-5}{2}\right) = -40$

$d = 0$

$n = 8$

51. $\sum_{k=1}^{10} k^2 = 1+4+9+16+25+36+49+64+81+100$

$S_{10} = 385$ Formula is $\frac{n(n+1)(2n+1)}{6}$

52. $\sum_{k=1}^{12} k = 1+2+3+\ldots+12$ Arithmetic

$a_1 = 1$ $\quad S_{12} = 12\left(\frac{1+12}{2}\right) = 78$

$a_{12} = 12$

$n = 12$

53. $\sum_{k=1}^{15}(-14+3k) = -11 + -8 + \ldots + 28 + 31$

$a_1 = -11$ $\quad S_n = 15\left(\frac{-11+31}{2}\right) = 150$

$a_{15} = 31$

$n = 15$

54. $\sum_{k=1}^{8}(4)^{k-1}$ geometric

$= 1 + 4 + 16 + 64 \ldots +$

$a_1 = 1$ $\quad S_8 = 1\left(\frac{1-4^8}{1-4}\right) = 21845$

$r = 4$

55. $\sum_{k=1}^{\infty} 9\left(\frac{1}{3}\right)^{k-1}$ geometric $= 9 + 3 + 1 + \ldots$

$r = \frac{1}{3}$ $\quad S = \frac{9}{1-\frac{1}{3}} = \frac{27}{2}$

$a_1 = 9$

56. $\sum_{k=15}^{825} 12 = \underset{a_{15}}{12} + 12 + 12 \ldots \cancel{+}12 + \underset{a_{825}}{12}$

$a_{15} = 12$

$a_{825} = 12$ $\quad S_{811} = 811\left(\frac{12+12}{2}\right) = 9732$

$n = 811$

57. $\sum_{k=1}^{18} k^2 = 1 + 4 + 9 \ldots + 18^2$

Formula

$\frac{n(n-1)(n-2)}{6} = \frac{18(18-1)(18-2)}{6} = 2109$

58. $\sum_{k=1}^{55} k = 1+2+3+\ldots+55$

$a_1 = 1$

$a_{55} = 55$ $\quad S_{55} = 55\left(\frac{1+55}{2}\right) = 1540$

$n = 55$

59. $\sum_{k=1}^{15} 18\left(\frac{1}{2}\right)^{k-1} = 18 + 9 + 4.5 + \ldots +$

$a_1 = 18$ $\quad S_{15} = 18\left(\frac{1-\left(\frac{1}{2}\right)^{15}}{1-\frac{1}{2}}\right) = 35.99$

$r = \frac{1}{2}$

60. $\sum_{k=1}^{7} -2(5)^{k-1}$

-39062

61. $\sum_{k=1}^{\infty} 8\left(\frac{4}{10}\right)^{k-1}$

$\frac{40}{3}$