2.2 Literal Equations and Formulas

2.2 OBJECTIVE

1. Solve a literal equation for a specified variable

Many problems in algebra require the use of **formulas** for their solution. Formulas are simply equations that express a relationship between more than one variable or letter. You are already familiar with a number of examples. For instance,

$$P = R \cdot B \qquad A = \frac{1}{2}h \cdot b \qquad P = 2L + 2W$$

are formulas for percentage, the area of a triangle, and the perimeter of a rectangle, respectively.

One useful application of the equation-solving skills we considered in Section 2.1 is in rewriting these formulas, also called **literal equations,** in more convenient equivalent forms.

NOTE A *literal equation* is any equation that involves more than one variable or letter.

Generally, that more convenient form is one in which the original formula or equation is solved for a particular variable or letter. This is called **solving the equation for a variable,** and the steps used in the process are very similar to those you saw earlier in solving linear equations.

Consider the following example.

Example 1

Solving a Literal Equation

Solve the formula

$$d = r \cdot t \qquad \text{for } t$$

This formula gives distance d in terms of a rate r and time t.

To solve for t means to isolate t on one side of the equation. This can be done by dividing both sides by r. Given

$$d = r \cdot t$$

we use the multiplication property of equations to divide by r, the coefficient of t.

$$\frac{d}{r} = \frac{r \cdot t}{r}$$

$$\frac{d}{r} = t$$

We usually write the equation in the equivalent form with the desired variable on the left. So

$$t = \frac{d}{r}$$

We now have t in terms of d and r, as required.

CHECK YOURSELF 1

Solve the formula $C = 2\pi r$ for r.

Solving a formula for a particular variable may require the use of both properties of equations, as the following example illustrates.

Example 2

Solving a Literal Equation

Solve the formula

$P = 2L + 2W$ for L

This formula gives the perimeter of a rectangle P in terms of its width W and its length L.

To solve for L, start by using the addition property of equations to subtract $2W$ from both sides.

NOTE We want to isolate the term with the variable we are solving for—here L.

$$P = 2L + 2W$$

$$P - 2W = 2L + 2W - 2W$$

$$P - 2W = 2L$$

We now use the multiplication property to divide both sides by 2:

$$\frac{P - 2W}{2} = \frac{2L}{2}$$

$$\frac{P - 2W}{2} = L$$

$$L = \frac{P - 2W}{2}$$

NOTE This result can also be written as

$L = \frac{P}{2} - W$

This gives L in terms of P and W, as desired.

CHECK YOURSELF 2

Solve the formula $ax + by = c$ for y.

You may also have to apply the distributive property in solving for a variable. Consider the following example.

Example 3

Solving a Literal Equation

Solve the formula

$A = P(1 + rt)$ for r

This formula gives the amount A in an account earning simple interest, with principal P, interest rate r, and time t.

First, we use the distributive property to remove the parentheses on the right.

$$A = P(1 + rt) = P + Prt$$

We now subtract P from both sides.

$$A - P = P - P + Prt$$

$$A - P = Prt$$

Finally, to isolate r, we divide by Pt, the coefficients of r on the right.

$$\frac{A - P}{Pt} = \frac{Prt}{Pt}$$

$$\frac{A - P}{Pt} = r$$

$$r = \frac{A - P}{Pt}$$

CHECK YOURSELF 3

Solve the equation for n.

$S = 180(n - 2)$

Often it is necessary to apply the multiplication property, to clear the literal equation of fractions, as the first step of the solution process. This is illustrated in Example 4.

Example 4

Solving a Literal Equation

Solve the formula for C.

$$D = \frac{C - S}{n}$$

This formula gives the yearly depreciation D for an item in terms of its cost C, its salvage value S, and the number of years n.

As our first step, we multiply both sides of the given equation by n to clear of fractions.

$$D = \frac{C - S}{n}$$

$$nD = n\left(\frac{C - S}{n}\right)$$

$$nD = C - S$$

NOTE On the *right* note that

$\frac{n}{n} = 1$

and multiplying by 1 leaves $C - S$.

We now add S to both sides.

$$nD + S = C - S + S$$

$$nD + S = C$$

$$C = nD + S$$

and the cost C is now represented in terms of n, D, and S.

CHECK YOURSELF 4

Solve the formula $V = \frac{1}{3}\pi r^2 h$ for h.

CHECK YOURSELF ANSWERS

1. $r = \frac{C}{2\pi}$ **2.** $y = \frac{c - ax}{b}$ **3.** $n = \frac{S + 360}{180}$ **4.** $h = \frac{3V}{\pi r^2}$

2.2 Exercises

In exercises 1 to 24, solve each of the formulas for the indicated variable.

1. $V = Bh$ for h

2. $P = RB$ for B

3. $C = 2\pi r$ for r

4. $e = mc^2$ for m

5. $V = LWH$ for H

6. $I = Prt$ for r

7. $V = \pi r^2 h$ for h

8. $S = 2\pi rh$ for r

9. $V = \frac{1}{3}Bh$ for B

10. $V = \frac{1}{3}\pi r^2 h$ for h

11. $I = \frac{E}{R}$ for R

12. $V = \frac{KT}{P}$ for T

13. $ax + b = 0$ for x

14. $y = mx + b$ for x

15. $P = 2L + 2W$ for W

16. $ax + by = c$ for y

17. $D = \frac{C - S}{n}$ for S

18. $D = \frac{R(100 - x)}{100}$ for R

19. $R = C(1 + r)$ for r

20. $A = P(1 + rt)$ for t

21. $A = \frac{1}{2}h(B + b)$ for b

22. $L = a + (n - 1)d$ for n

23. $F = \frac{9}{5}C + 32$ for C

24. $C = \frac{5}{9}(F - 32)$ for F

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ANSWERS

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Solve each of the following exercises using the indicated formula from exercises 1 to 24.

25. A rectangular solid has a base with length 6 cm and width 4 cm. If the volume of the solid is 72 cubic centimeters (cm^3), find the height of the solid. See exercise 5.

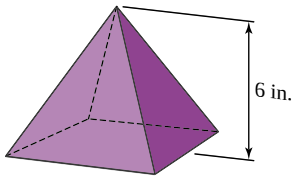
26. A cylinder has a radius of 4 inches (in.). If its volume is 144π cubic inches ($in.^3$), what is the height of the cylinder? See exercise 7.

27. A principal of $2000 was invested in a savings account for 4 years. If the interest earned for that period was $480, what was the interest rate? See exercise 6.

28. The retail selling price of an item, R, was $20.70. If its cost, C, to the store was $18, what was the markup rate, r? See exercise 19.

29. The radius of the base of a cone is 3 cm. If the volume of the cone is 24π cm^3, find the height of the cone. See exercise 10.

30. The volume of a pyramid is 30 $in.^3$. If the height of the pyramid is 6 in., find the area of its base, B. See exercise 9.



31. If the perimeter of a rectangle is 60 ft and its length is 18ft, find its width. See exercise 15.

32. The yearly depreciation, D, for a piece of machinery was $1500 over 8 years. If the cost of the machinery was $15,000, what was its salvage value, S? See exercise 17.

33. A principal of $5000 was invested in a time-deposit account paying 9% annual interest. If the amount in the account at the end of a certain period was $7250, for how long was the money invested? See exercise 20.

34. The area of a trapezoid is 36 $in.^2$. If its height is 4 in. and the length of one of the bases is 11 in., find the length of the other base. See exercise 21.

Answers

1. $h = \frac{V}{B}$ **3.** $r = \frac{C}{2\pi}$ **5.** $H = \frac{V}{LW}$ **7.** $h = \frac{V}{\pi r^2}$ **9.** $B = \frac{3V}{h}$

11. $R = \frac{E}{I}$ **13.** $x = -\frac{b}{a}$ **15.** $W = \frac{P - 2L}{2}$ **17.** $S = C - nD$

19. $r = \frac{R - C}{C}$ **21.** $b = \frac{2A - hB}{h}$ **23.** $C = \frac{5}{9}(F - 32)$ or $C = \frac{5F - 160}{9}$

25. 3 cm **27.** 6% **29.** 8 cm **31.** 12 ft **33.** 5 years